Journal of Science-FAS-SEUSL (2024) [Volume 05 No.1]

ISSN:2738-2184

ANALYTICAL SOLUTIONS OF ONE-DIMENSIONAL HEAT EQUATION USING SEPERATION OF VARIABLES METHOD

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Abstract

The heat equation is a crucial partial differential equation that defines how heat is distributed in a particular location over time. It has many fundamental importance in diverse science and engineering fields. In this paper, the one-dimensional heat equation in a thin rod subject to the Dirichlet boundary condition with distinct initial conditions has been solved by analytical methods using the separation of variables and Fourier's techniques. Each diverse initial condition was taken as a test example and solved. It was found that it provided distinct general solutions as analytical solutions. To analyze further, we have used the MATLAB graphical representation of the Fourier's coefficient and the approximative behavior of the selected initial condition for 5 iterations of the general solutions obtained. These plots show the idea behind the theory of Fourier series representation in the analytical solutions, which tend to coincide with the initial conditions when increasing the number of iterations. Finally, we have concluded that obtaining these analytical solutions are not always an easy task, and it is still difficult to generalize such results when new problems arise. In addition, there are many limitations and complexity issues with the analytical method for boundary value problems. So, the analytical solutions are most sought after given their accuracy and usefulness in validating further numerical methods.

Keywords: One-dimensional heat equation, Analytical solution, Separation of variables method, Fourier's techniques.

1. Introduction

Mathematics plays an important role in everyday life. Researchers employ mathematical concepts to comprehend patterns, create relationships, predict the answers to specific issues, and make better logic-based decisions, among other things.

Differential equations play an important role in many fields of mathematics, including engineering, physics, biology, and economics [1]. A differential equation connects the derivatives of one or more unknown functions. In most applications, functions are used to represent physical quantities, and derivatives are used to represent their rates of change. Differential equations can be divided into two major types, such as Ordinary Differential Equations (ODE) and Partial Differential Equation (PDE), which can be further divided into linear and non-linear, homogeneous and non-homogeneous, etc.

Partial Differential Equations (PDEs) are mathematical equations that are useful for representing natural physical events. They are used to mathematically formulate the physical phenomena and find

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the solution to the modeled problems involving functions of several variables, such as the propagation of heat or sound, fluid flow, elasticity, electro-statics, and electro-dynamics, [2].
The second-order linear PDE with two independent variables has the form:

$$A\frac{\partial^2\phi}{\partial x^2} + 2B\frac{\partial^2\phi}{\partial x\partial y} + C\frac{\partial^2\phi}{\partial y^2} + D\,\frac{\partial\phi}{\partial x} + E\,\frac{\partial\phi}{\partial y} + F = 0 \quad \_\_\_\_\_\_\_\_\_\_(a)\,.$$

Here $\phi(x, y)$ is the real-valued function of two independent variables x and y. Usually, x represents one-dimensional position and y represents time. If $F = 0$ then the equation (a) is a homogeneous equation, and A, B, C, D, E and F are called the coefficients of the equation.

The linear PDEs are distinguished from a simpler equation that has the basic type called parabolic, if $B^2 - AC = 0$. The solutions of this type have their own characteristic qualitative differences. The name parabolic is used since the assumptions of the coefficients used in equation (a) are the same as the analytic geometric equation to define a planar parabola as,

$$Ax^2 + 2Bxy + Cy^2 + Dx + E\,y + F = 0\,.$$

Heat is an energy that transfers from one medium to another, from a hot region (higher temperature) to a colder region (lower temperature). According to the medium of the conductor the heat takes different forms, such as being transferred in solids by the process of conduction, in liquids and gases, by the process of convection, and as radiation in electro-magnetic waves. The heat equation in one-dimensional space is a kind of PDE that explains the heat distribution over a period of time in a solid medium. In the early 1800s, a mathematical study of heat and the development of the theory of heat conduction were analyzed [3]. The diffusion equation is a more generic variant of the heat equation that develops in the context of random chemical temperature fluctuations and other processes. A deeper understanding of heat flow has many significant implications for science and industry.

Tadmar. E, in his paper [4], says partial differential equations provide a quantitative explanation for many important models in the physical, biological, and social sciences. Basically, the heat equation is always applied to the physical heat flow mechanisms. It is important to derive the problem and get the model of the heat equation with the boundary and initial conditions correct. Then, the solution for the equation can be done by using analytical methods such as separation of variables, the D-Alembert Method, or others.

Analytical and numerical solution methods for a PDE have gained recent interest among researchers for finding approximate solutions. This interest has been driven by the high demand for applications that investigate the analytical and numerical methods for solving initial and boundary value problems. By changing the initial and boundary conditions of the particular PDE, we will have different solutions, which is an interesting factor in understanding the physical behavior of the problem. These methods also allow the mathematical recreation of the physical processes that frequently appear in science and engineering. The study of [5] provides the detailed process of the one-dimensional heat diffusion equation, which represents the heat distribution in a particular region and offers the basic tool for heat conduction analysis. Analytical and numerical solutions of one-dimensional heat equations have been studied by many scholars. The authors of [6] have studied the analytical solution of the heat equation by using the separation of variable methods, the Fourier transform method, and Laplace transform method. Also, in [7], Norazlina at el, present the analytical solution of the homogeneous 1D heat equation with Neumann boundary conditions. Gorguis and Benny Chan examined a

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comparative study between the traditional separation of variables method and the Adomian method for heat equations [8].

Simulation processes are an effective tool in applied sciences to find solutions and predict the behaviors of the physical problem we are interested in comparing with existing models under different conditions. These require careful planning of the mathematical models of PDE with proper initial and boundary conditions and the algorithms to solve the model using the computational tools [3]. In industry and the real world, the problem will consist of more than one variable, and it is not always easy to solve the problem using manual calculations. So, to get the solutions faster and more accurately, we can use mathematical software with fast algorithms. In this research, we use MATLAB programming since it is easier to generate the codes [10].

The aim of this paper is to get a deeper insight into one-dimensional heat diffusion equations on a thin rod. The modeled equation of the one-dimensional heat equation with Dirichlet boundary conditions and different initial conditions for trigonometric, polynomial, exponential, and piecewise functions is solved. The separation of variables method and Fourier's techniques have been adapted to get the general analytic solutions of the test examples. Finally, the graphical representation of Fourier coefficients and the approximate behavior of the selected initial condition for 5 iterations of the general solutions obtained using MATLAB software to interpolate the results.

2. Methodology

Consider the model of the one - dimensional homogeneous heat diffusion equation along the thin rod bar of length L in Figure1 is

$$\frac{\partial U}{\partial t} = \alpha \frac{\partial^2 U}{\partial x^2}, \quad \text{for} \quad 0 < x < L, \quad t > 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (1)$$

subject to the homogeneous boundary conditions (BC):

$$U(0,t) = 0 \quad \text{and} \quad U(L,t) = 0, \qquad t > 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (2)$$

and initial condition (IC):

$$U(x,0) = f(x), \qquad 0 < x < L, \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (3)$$

where $U(x,t)$ is the temperature at space x and time t, α is the thermal diffusivity of the rod.

Figure 1: Coordinate system of the rod

The analytical solution method is based on finding a change of variable to transform the equation into something soluble and is also used to frame the PDE problems in a well-understood form and obtain the exact solution.

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In this section, we derive the general solution of the above model using the separation of variables method, and Fourier analysis techniques. The separation of variables method is a powerful method, that can be applied to solve linear PDEs. The method reduces the PDE into a system of two ODEs.

Obtaining the analytical solution to the one-dimensional heat equation model involves three steps:

Step 1: Apply separation of variables to the problem and obtain two ODEs,

Step 2: Determine the solutions that satisfy the given boundary conditions,

Step 3: Superposition of Fourier techniques used to obtain the solution that satisfies the given initial conditions.

First assume that the solution of the above model, $U(x,t)$ is of the separable form

$$U(x\,,t) = X(x)\,.T(t) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_} \; (2.1)$$

where, $X(x)$ and $T(t)$ are the separated solutions of the PDE problem.

Now applying the boundary conditions given in the equation (2) of the model, on the equation (2.1), when $x = 0$, and $x = 0$ we get

$$U(0\,,t) = 0 \;\Rightarrow\; X(0)\,.T(t) = 0\;, \qquad \forall t \;,$$

and

$$U(L\,,t) = 0 \;\Rightarrow\; X(L)\,.T(t) = 0\;, \qquad \forall t \;,$$

These results implies

$$X(0) = 0 = X(L)\,. \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_} \; (2.1-\text{II})$$

Substituting equation (2.1) into the given PDE equation (1), we get

$$\frac{\partial}{\partial t}\big(X(x)\,.T(t)\big) = \alpha\,.\,\frac{\partial^2}{\partial x^2}\big(X(x)\,.T(t)\big)$$

which gives, on simplification,

$$\mathrm{X(x)}\,.\,T'(t) = \alpha\,.\,\mathrm{X''(x)}\,.\,T(t) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_} \; (2.2)$$

where, prime (‘) denotes derivative w.r.t the respective variable.

To separate the variables, divide the equation (2.2) by $\alpha\,.X(x)\,.T(t)$, which gives

$$\frac{T'(t)}{\alpha\,T(t)} = \frac{X''(x)}{X(x)}\,. \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_} \; (2.2a)$$

Note that the left-hand side is function of t only and right-hand side is function of x only. This is possible if both are equal to a constant, say, k

$$\frac{T'(t)}{\alpha\,T(t)} = \frac{X''(x)}{X(x)} = k\;,\;\; k \text{ is a constant,} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_} \; (2.2b)$$

Then we can have two ordinary differential equations given as

$$\frac{T'(t)}{\alpha\,T(t)} = k \;\Rightarrow\; T'(t) = \alpha\,k\,T(t) \;\Rightarrow\; T'(t) - \alpha\,k\,T(t) = 0\,, \text{\_\_\_\_\_\_\_\_\_\_\_\_\_} \; (2.2c)$$

and

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ISSN:2738-2184

$$\frac{X''(x)}{X(x)} = k \;\Rightarrow\; X''(x) = k\,X(x) \;\Rightarrow\; X''(x) - k\,X(x) = 0\,. \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2.2d)$$

Consider the solutions of the equation (2.2d) depend on the sign of k. These signs can be defined in three cases as $k > 0$, $k = 0$ $and\; k < 0$ which can be analyzed as

Case I: for the sign of $k > 0$, suppose that $k = \lambda^2$; $\lambda > 0$, then we have

$$X''(x) - \lambda^2\,X(x) = 0\,. \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2.2e)$$

From the Table 1 results, the solutions of the equation (2.2e) will be

$$X(x) = Ae^{\lambda x} + Be^{-\lambda x} \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2.2f)$$

When we applying the boundary conditions of equation (2.1 – II) on equation (2.2f), we can obtain

$$X(0) = 0 \quad \Rightarrow\; 0 = A + B \;\Rightarrow\; B = -A$$

and $$X(L) = 0 \;\Rightarrow\; 0 = Ae^{\lambda L} + Be^{-\lambda L}\,,\; \lambda > 0\,,\; L > 0\,,$$

$$\Rightarrow\; 0 = Ae^{\lambda L} - Ae^{-\lambda L} \quad 0 = A(e^{\lambda L} - e^{-\lambda L})$$

clearly $\left(e^{\lambda L} - e^{-\lambda L}\right) > 0$, $\lambda > 0$, $L > 0$, which implies $A = 0 = B$.

This is obviously a trivial solution.

Case II: for the sign of $k = 0$, we have

$$X''(x) = 0\,. \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2.2g)$$

From the Table 1 results, the solutions of the equation (2.2e) will be

$$X(x) = Ax + B \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2.2h)$$

When we applying the boundary conditions of equation (2.1 – II) on equation (2.2h), we can obtain

$$X(0) = 0 \quad \Rightarrow\; 0 = A(0) + B \;\Rightarrow\; B = 0$$

and $$X(L) = 0 \;\Rightarrow\; 0 = A(L) + B\,,$$

$$\Rightarrow\; 0 = A.L\,, \quad \text{since} \quad L > 0 \;\Rightarrow A = 0\,,$$

which implies $A = 0 = B$. This is also a trivial solution.

Case III: for the sign of $k < 0$, suppose that $k = -\lambda^2$, then we have

$$\frac{T'(t)}{\alpha T(t)} = \frac{X''(x)}{X(x)} = -\lambda^2 \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2.3)$$

Now to get the non-trivial solutions on this case, we need to analyze the two ODEs derived from equation (2.3) as

$$T' = -\alpha\,\lambda^2\,T \quad and \quad X'' = -\lambda^2\,X \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2.4)$$

Using the **Table 1** we can obtain the general solutions of the equation (2.4)

$$T(t) = Ce^{-\alpha\lambda^2 t} \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2.5)$$

and

$$X(x) = A\cos\lambda x + B\sin\lambda x \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2.6)$$

Applying the boundary conditions given in the equation (2.1 - II), on the equation (2.6), we get

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$$X(0) = 0 \Rightarrow 0 = A(1) + B(0) \quad , \quad \text{since} \quad \cos 0 = 1 \quad \text{and} \quad \sin 0 = 0$$

which gives $A = 0$, and

$$X(L) = 0 \Rightarrow 0 = A\cos\lambda L + B\sin\lambda L \quad \Rightarrow B\sin\lambda L = 0 \quad ,$$

which gives $\lambda^2 = \frac{n\pi}{L}$, $n \in \aleph$, is called as eigen-values of the problem.

Differential Equation Types	General solution
$y'' + \lambda^2 y = 0$	$y(x) = A\cos\lambda x + B\sin\lambda x$
$y' = ky$	$y(t) = pe^{kt}$
$y'' - \lambda^2 y = 0$	$\mathrm{y(x)} = Ae^{\lambda x} + Be^{-\lambda x}$

Table 1: General solutions of the ODEs.

Now, up to the constant multiples, the only solutions to the model is

$$X_n(x) = B_n \, . \, \sin\left(\frac{n\pi x}{L}\right),$$

is called as eigen-functions of the problem, and

$$T_n(t) = C_n \, . \, e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t} .$$

We thus have the normal modes of the heat equation, by the equation (2.1) as

$$U_n(x\,,t) = X_n(x) \, . \, T_n(t)$$

or

$$U_n(x\,,t) = b_n \, . \, \sin\left(\frac{n\pi x}{L}\right) . e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t} .$$

Here

$$b_n = B_n \, . \, C_n \quad \text{and} \quad n \in \aleph .$$

Applying the principle of superposition, the general solution of the model of the one-dimensional heat equation (1) – (3) becomes

$$U(x\,,t) = \sum_{n=1}^{\infty} U_n(x\,,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) . e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t} \quad \_\_\_\_\_\_\_\_\_\_ (A)\,,$$

when we impose our initial condition, we get

$$f(x) = U(x\,,0) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right).$$

Now consider,

$$\int_0^L f(x) . \sin\left(\frac{m\pi x}{L}\right) dx = \int_0^L \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) . \sin\left(\frac{m\pi x}{L}\right) dx$$

$$= \sum_{n=1}^{\infty} b_n \int_0^L \sin\left(\frac{n\pi x}{L}\right) . \sin\left(\frac{m\pi x}{L}\right) dx$$

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Using the orthogonality of sine functions gives,

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) . \sin\left(\frac{m\pi x}{L}\right) dx = \begin{cases} \frac{L}{2}, & if \quad m = n; \\ 0, & if \quad m \neq n. \end{cases} \quad = \quad \frac{L}{2}\,\delta_{mn} .$$

So,

$$\int_0^L f(x) . \ \sin\left(\frac{m\pi x}{L}\right) dx = \sum_{n=1}^{\infty} b_n . \frac{L}{2}\,\delta_{mn} = \frac{L}{2}\, b_m ,$$

which is the Fourier sine series expansion of $f(x)$. Hence, we have

$$b_m = \frac{2}{L}\int_0^L f(x) . \ \sin\left(\frac{m\pi x}{L}\right) dx \ , \qquad \mathrm{m} \in \aleph. \qquad \_\_\_\_\_\_\_\_\_\_ (B)$$

3. Results and Discussion

The analytical solutions of the test examples with different initial conditions of $f(x)$ with the one-dimensional heat model of equations (1) – (3) has obtained using the separation of variables and Fourier series techniques. The analytical solutions and the related graphical representations have been derived by the MATLAB software to understand the phenomena of the given physical problems.

Test Example 01: $f(x) = \sin\left(\frac{\pi x}{L}\right)$

Consider the model of the one-dimensional heat equation (1) – (2) and the initial condition:

$$U(x\,,0) = \sin\left(\frac{\pi x}{L}\right) \ , \qquad 0 < x < L \quad \_\_\_\_\_\_\_\_\_\_ (4)$$

Substituting the IC (4) into the equation and using the orthogonality of the sine functions, we have

$$b_m = \frac{2}{L}\int_0^L \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = \begin{cases} 1, & if \ m = 1, \\ 0, & otherwise \end{cases} .$$

That is, $b_1 = 1$ and $b_m = 0\,, \quad m = 2, 3, \ldots$

Now, the equation of (A) gives the exact solution as

$$U(x\,,t) = \sum_{n=1}^{\infty} U_n(x\,,t) = \sum_{n=1}^{\infty} b_n \ . \ \sin\left(\frac{n\pi x}{L}\right) . \, e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t} ,$$

Then with the IC, $f(x) = \sin\left(\frac{\pi x}{L}\right)$, it becomes

$$U(x\,,t) = \sin\left(\frac{\pi x}{L}\right) . \, e^{-\alpha\left(\frac{\pi}{L}\right)^2 t} .$$

Test Example 02: $f(x) = -x^2 + x$

The model of the one-dimensional heat equations (1) and (2) with initial condition:

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$$U(x,0) = -x^2 + x \ , \qquad 0 < x < L \quad \_\_\_\_\_\_\_\_\_\_ (5)$$

Using the equation (B) and substituting the IC (5) into the equation, we get

$$b_m = \frac{2}{L}\int_0^L (-x^2 + x) \ . \sin\left(\frac{m\pi x}{L}\right) dx$$
$$= -\left(\frac{4\left((-1)^m - 1\right)}{(m\pi)^3}\right) \ , \quad m \in \aleph$$

So, the equation of (A) gives

$$U(x,t) = \sum_{n=1}^{\infty} -\left(\frac{4\left((-1)^m - 1\right)}{(m\pi)^3}\right) . \sin\left(\frac{n\pi x}{L}\right) . e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t} .$$

Test Example 03: $f(x) = e^x$

In a similar way as above examples, The one-dimensional heat model of equations (1) and (2) with the initial condition:

$$U(x,0) = e^x \ , \qquad 0 < x < L \quad \_\_\_\_\_\_\_\_\_\_ (6)$$

The equation (B) with the IC (6) gives,

$$b_m = \frac{2}{L}\int_0^L e^x . \ \sin\left(\frac{m\pi x}{L}\right) dx = \frac{2\pi m\left((-1)^m e - 1\right)}{(m\pi)^2 + 1} \ , \qquad m \in \aleph.$$

So, the equation of (A) gives the exact solution as

$$U(x,t) = \sum_{n=1}^{\infty} \left(\frac{2\pi n\left((-1)^n e - 1\right)}{(n\pi)^2 + 1}\right) . \sin\left(\frac{n\pi x}{L}\right) . e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t} .$$

Test Example 04: $f(x) = \begin{cases} x \ , & x < \frac{L}{2} \\ x - \frac{L}{2} \ , & x > \frac{L}{2} \end{cases}$

The model of the one-dimensional heat equations (1) and (2) with IC:

$$U(x,0) = f(x) = \begin{cases} x \ , & x < \frac{L}{2} \\ x - \frac{L}{2} \ , & x > \frac{L}{2} \end{cases} \ , \qquad 0 < x < L \quad \_\_\_\_\_\_\_\_\_\_ (7)$$

Again, using the equation (B) and substituting IC (7), we get

$$b_m = -\left(\frac{(-1)^{\frac{m}{2}}\left[(-1)^{\frac{m}{2}} - 1\right]^2}{2m\pi}\right) \ , \quad m \in \aleph.$$

Then the equation of (A) gives the exact solution of

$$U(x,t) = \sum_{n=1}^{\infty} -\left(\frac{(-1)^{\frac{m}{2}}\left[(-1)^{\frac{m}{2}} - 1\right]^2}{2m\pi}\right) . \ \sin\left(\frac{n\pi x}{L}\right) . e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t} .$$

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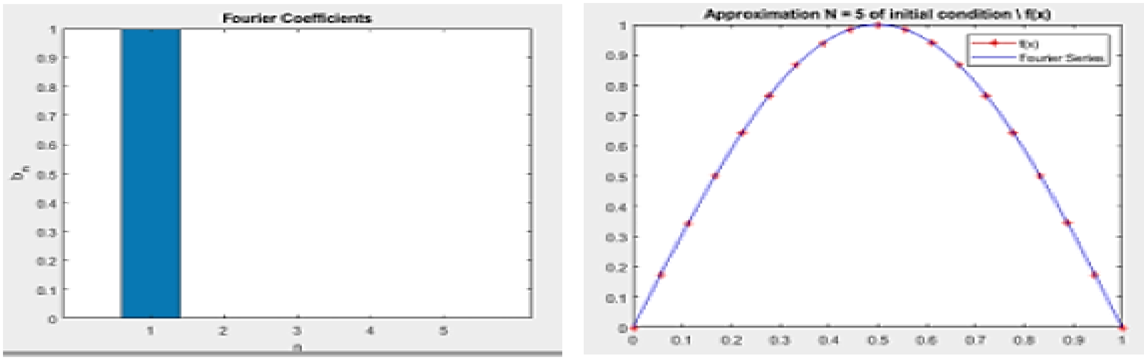
The graphical representation of Fourier's coefficient b_m for the values of m from 1 to 5 and the approximative interpretation of the initial condition with the Fourier's coefficients at the end of the fifth iteration of the Test examples 01 – 04 are plotted in Figure 2(a) – (d), respectively.

Here, we have used MATLAB to plot the graphical representation views. These plots show the idea behind the theory of Fourier series representation in the analytical solutions, which tend to coincide with the initial conditions when increasing the number of values m.

Note that the blue rectangular plots on each left figure show the value of the Fourier's coefficient of equation (B) at the values of $m = 1$ to $m = 5$. We can notice in Figure 2(a), in Test example 01 for the initial condition $f(x) = \sin\left(\frac{\pi x}{L}\right)$, the values are $b_1 = 1$ and $b_m = 0$, $m = 2, 3, 4.5$. Therefore there is only one blue rectangular bar of value $b_1 = 1$ and empty slots for other values of m. Similarly, Figure 2(b) – (d) shows the Fourier coefficients b_m for $m = 1$ to $m = 5$ values of Test examples (2) – (4) in the left Figures, respectively.

Likewise, in the right plots of Figure 2(a) – (d), the red dot line indicates the IC of each Test example, and the blue lines represent how the Fourier coefficients b_m coincide with the IC at the end iteration value of $m = 5$. We observed that, since the general solution (A) sums up to infinity values, when increasing the m values in the right plot, both red and blue plots become more coincident for all Test examples (1) – (4).



a)

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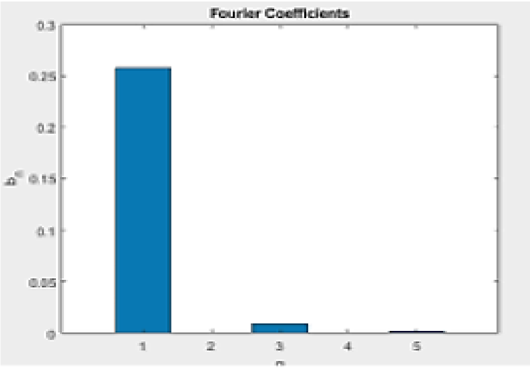
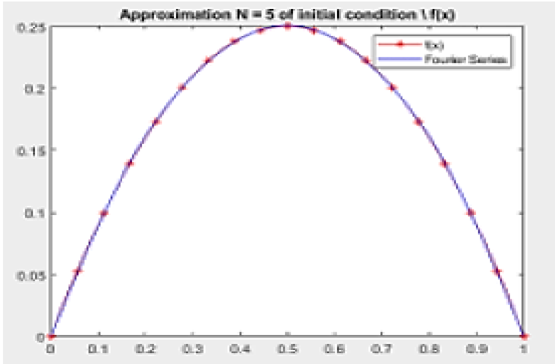
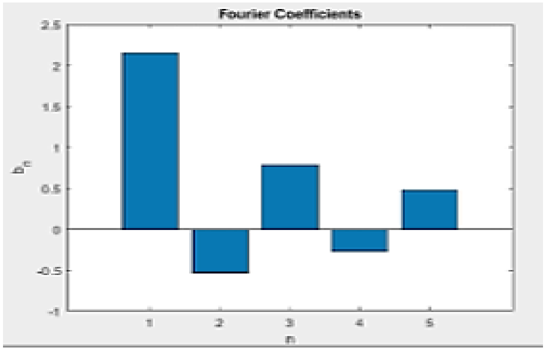
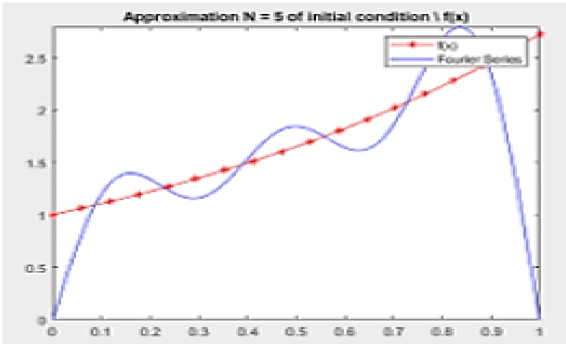
ISSN:2738-2184

b)

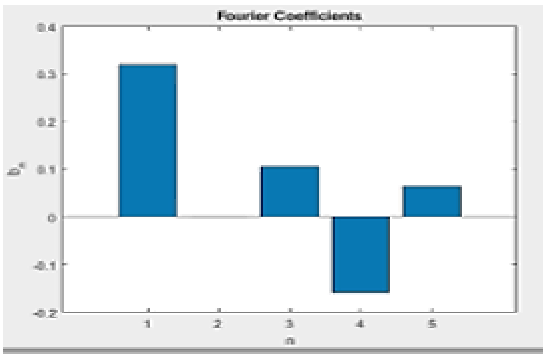


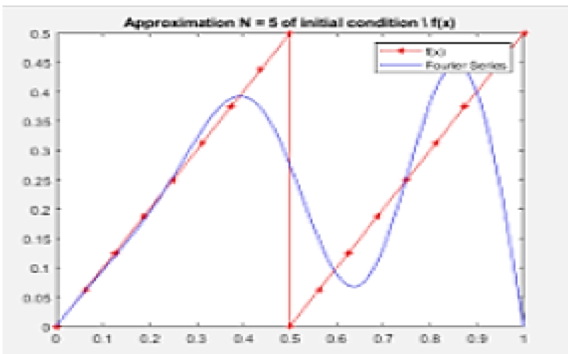






c)





d)

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Figure 2(a) – (d): Graphical representation of the Fourier's coefficient b_m with the IC of equations (4) – (7), respectively.

04 Conclusion

In this study, we have analyzed how the analytical solutions of the one-dimensional heat equation are influenced by the different initial conditions. We also examined the relationship between the initial condition and the Fourier coefficient b_m using the graphical representations. The model problems were evaluated with a set of fixed Dirichlet boundary conditions and diverse initial conditions as test examples to confirm the hypothesis.

However, obtaining these analytical solutions are not always an easy task, and it is still difficult to generalize such results to new problems that arise. In addition, there are many limitations and complexity in the analytical method for such problems. So, the analytical solutions are most sought after given their accuracy and usefulness in validating further numerical methods. Which can be analyzed in future works on this study's problems.

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